Common Brain Teasers: Sometimes, you will encounter brain teasers as a part of your interview. Do not panic, as the interviewer is more so interested in how you explain your thinking and thought process more than anything

1. You get paid the equivalent number of dollars for the number you roll on a dice (i.e., $\$ 4$ if you roll a 4). How much are you willing to pay for this roll?
a. This is a question about expected value, which is the sumproduct of all the possibilities and their probabilities (i.e. ( 1 * $(1 / 6)$ ) + ( 2 * $(1 / 6))+\left(3^{*}(1 / 6)\right)$, and so on until you get to 6 , the max number of possibilities). This equates to $\$ 3.50$. A faster way to approach this is realizing that each outcome has the same probability, so the weight is equal across all outcomes -- hence, you can just average 1 through 6, which is $\$ 3.50$
b. The interviewer follows up and asks, "How much would you pay to roll again?"... you should only roll again if the value obtained is lower than the expected value of $\$ 3.50$ (i.e. if you roll a $1-3$ ). Because you get to roll again, the price to play this game logically should be higher. Strategy should not change... probability of rolling a 1 through 6 has not changed, so you can just take an average of all of the outcomes should you decide to play again. Hence, it would be the average of $3.5,3.5,3.5,4,5$, and 6 , as you would only roll again for three of the outcomes (expected value of the game of $\$ 3.50$ ) and wouldn't roll again if you obtained a 4, 5, or 6 . Hence, the price you would be willing to pay to roll a second time would be $\$ 4.25$
c. The interviewer is persistent... would you roll a third time? The expected value of the game rolling a second time is $\$ 4.25$, meaning you would only play again if you rolled less than a 4 . Using the same strategy, because the probabilities haven't changed, it is the average of $4.25,4.25,4.25,4.25,5$, and 6 . Hence, the price you would be willing to pay to roll a third time is $\$ 4.67$... for a $5^{\text {th }}$ role, $\$ 4.94$, getting closer to $\$ 6$ as the number of rolls keeps on increasing
2. What is the probability of drawing 3 8's in a card deck?
a. Good rule to know: There are 52 cards in a deck and each card repeats itself 4 times across the suits (hearts, clubs, diamonds, and spaded).
b. Probability of drawing 18 is $4 / 52 \ldots$ probability of then drawing the $2^{\text {nd }}$ one consecutively is $3 / 51 \ldots$ then $2 / 50$, etc. these fractions simplify to $1 / 13,1 / 17$, and $1 / 25$, respectively. The probability of drawing three 8 's in a card deck, in a row, is the probabilities multiplied against each other... i.e. 13 * 17 * 25.13 * 17 can be thought of as 17 * $10+3$ * 17 , which is $170+51$, or 221 . 221 multiplied by 25 is not the most intuitive, so break 25 down into its prime factors ( 5 and 5 ). Hence, 221 * 5 is $1,105, ~ 5$ is 5,525. The answer is $1 / 5,525$. Disclaimer: The interviewer will most likely ask you the probability of drawing two cards, not three, given the numbers can get very big. However, it is good to practice for harder types of questions!
3. A room has three light bulbs. There are three light switches outside the door for each light bulb. If you can only enter the closet one time, how do you determine which switch controls which light bulb?
a. Turn two switches on and wait for a while (to let them heat up). Turn one switch off. Enter the room. The one that is turned on corresponds to the remaining light switch that is switched on, the one that feels warm corresponds to the switch that you turned off, and the remaining bulb corresponds to the switch that was never touched
4. A lily pond doubles every minute. After an hour, the lily fills the entire pond. When is it $1 / 8$ full?
a. The key here is to work backwards, not from the very first minute. At 59 minutes, the lily pond was half full. At 58 minutes, it is $1 / 4$ full. And at 57 minutes, the lily pond is $1 / 8$ full, dividing each time by a half
5. You have a $10 \times 10 \times 10$ cube made up of $1 \times 1 \times 1$ smaller cubes. The outside of the cube is completely painted. On how many of the smaller cubes is there any paint?
a. The easier way to address this is to figure out the number of cubes that are not painted. Hence, $8 \times 8 \times 8$ cubes are not painted (as a cube on each end of the row has at least one side painted). This equates to 64 * 8 , which is 512 . Then, take $1,000-512$, which is 488 cubes.
b. If your interviewer tells you to solve it "the other way" (i.e. counting those that are painted), realize that there are two $10 \times 10$ sides (200) + two 10 * 8 sides (160) + two 8 * 8 sides (128) that are painted, resulting in 488 cubes that are painted
c. Take the intuitive approach (i.e., a) first!
